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Ecological Approach to Economic Systems

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ABSTRACT

Economic system has the same multilateral structure as that of ecological one. There are four different levels - individual, population, community, and ecosystem levels, in both systems. In addition to this, functional behaviors of economic system are similar to those of ecological system. Because of these similarities we can analyze the properties of economic system based on the theory of ecological one, which made great progess recently. In this paper the mathematical models are given which can describe behaviors of economic system. These are logistic, replicon, and competition models; which are borrowed from the fields of evolutionary ecology and population genetics. Replicon model is applied to studying the evolving process of technologies. Competition model is used to describe the relationship between consumers and producers in a market. These models can clarify the underlying mechanisms of economic system.

1. INTRODUCTION

The behavior of economic system is similar to that of ecological one in many points. Every commodity shows time dependent behavior in a market, which is sometimes similar to a life cycle of biological individual. The growth process of a new technology can be understood by the logistic curve, which describes the growth process of a biological species.

Industrial structure shows time evolution, which can be related with the change of landscape of ecological community known as the ecological succession.

From the structural viewpoint, ecological system shows multilateral structure. There are four different levels in the system. They are individual level, population level, community level, and ecosystem level. Here, population means an assemblage of individuals which belong to a common species and live together in the same region. Community represents a set of biological species which live together in the same region and interact with each others. Ecosystem refers to a collection of community and its physical environment.

Similar to the ecological system, economic system showes multilateral structure. As an example let us consider the production sector of the economic system. Corresponding to the ecological system we can distinguish four different levels. The first is an individual firm or company. The second is each industry which consists of many firms producing the same commodity. This corresponds to a population. The third is the assemblage of each industry, which is nothing but the whole industries. And fourth is a set of whole industries and itheir surrounding consumers and physical environment.

Because of these similarities we can analyze the properties of economic system based on the theory of ecological one.

From the functional point of view, ecological and economic systems undergo evolutionary and adaptive processes. As a result of evolutionary process the landscape of ecological and economic systems change temporally. Generally speaking, there are two elements of motive force in evolutionary process: one is the phylogenetic inertia of each population, and the other is selection pressures from its surrounding environment. In other words, each population evolves under the influence of selection pressures within the constraint of its phylogenetic inertia.

To study in more detail the economic system from the ecological viewpoint, it is necessary to build mathematical models of the economic system, which are related with those of ecological one. In the next chapter we will give the mathematical models which can describe both of ecological and economic systems. Chapter 3 is concerned with the application of replicon model to the evolving process of technologies in an industry. Chapter 4 discusses competition among commodities, which can be described by a generalized Lotka-Volterra model. The models given in these two chaptes are most important ones used in the ecological system. Discussions are given in the final chapter.

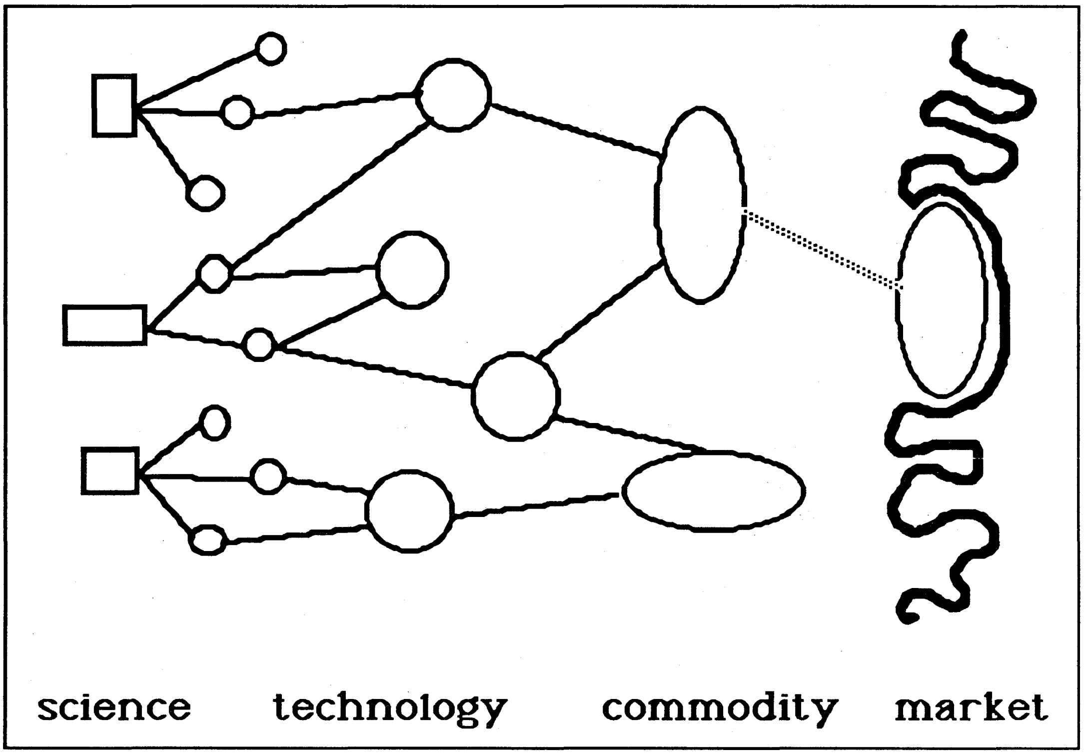
2. ECOLOGICAL MODELS OF ECONOMIC SYSTEMS

From the ecological point of view, economic system can be understood as interacting processes of different "economic species." Economic system produces plenty of commodities using resouce, capital, and labor as the factors of production. During production process technologies play main role, which are supported directly or indirectly by scientifc activity. Then we can say that economic species consist of science, technology, commodity, and market. In a market there are consumers and producers. Interaction among these elements can be illustrated by Figure 1. Similar figure is given by Clark and Juma (1987).

Scientific activity produces new findings about nature and gives novel materials. Technology integrates these products and fruits of scientific activity in order to produce new commodities. As a result of these activities plenty of commodities appear in the market. And there occurs competition among commodities.

Generally each element indicated in Figure 1 can be considered as a replicon. Here replicon means any replicating unit having internal states (Matsuda and Ishii, 1980). And economic processes can be described by birth, growth, and death of defferent kind of replicons.

FIGURE 1. ECOLOGY OF ECONOMIC PROCESS



Following the above discussion, we can use the formulation of evolutionary ecology to describe economic process theoretically. Essentially there are two basic theoretical models in the fields of evolutionary ecology and population genetics.

One is the replicon model, which describes the evolutionary process of replicons. In the next section we will apply this model to study the innovative and imitative processes of technologies. Replicon model includes logistic model as its special case, which represents growing process of a replicon.

The other is the Lotka-Volterra model, which describes the competitive, cooperative, and predator-prey processes among replicons. We will apply this model to describe the competitive interaction between commodities in a market.

Since economic processcan be represented by the dynamic process of replicons, we can use succesfully these biological models to describe economic processes. These models can be summarized in Table 1.

TABLE 1. ECONOMIC SYSTEM VS BIOLOGICAL SYSTEM

economic system	biological system	model
growth process of a technology	growth process of a population	logistic model
competition among goods	competition in a community	Lotka-Volterra
evolving process of technologies	evolving process of genes	replicon model

Here, the growing process of a new technology is usually represented by the logistic model, which describes the growth process of a biological population. The competitive interactions of firms can be expressed by the Lotka-Volterra model, which represents the ecological predator-prey interaction, or the competitive interactions of populations using the same resources. The replicon model can describe evolving process of technologies as well as of genes.

3. THE REPLICON MODEL OF TECHNOLOGIES

We consider firms as replicons, since firms can be thought as replicating units with technological levels as their essential internal states. Here replication of a firm means the change in size of the firm in unit time. Then we can apply the replicon model to studying the evolutionary processes of firms. We consider an assemblage of firms that produce a single homogeneous product; that is, they belong to the same industry.

A. General Formulation

The behavior of firm can be represented by a production function, which relates the amount of product output to related input factors. We take capital and labor as two input factors, following the standard economic approach. We denote the size of capital, labor and product as K, L, and Q, respectively. Then the production function can be described generally as

$$Q = f(\alpha K, \beta L) \tag{1}$$

where α and β denote, respectively, the technological coefficients of capital and labor.

The value of the coefficients α and β can change as a result of technological innovation and imitation. For simplicity's sake we assume that the coefficients α and β can be classified by discrete series as follows:

$$\alpha = (\alpha_1, \alpha_2, \dots, \alpha_i, \dots) \tag{2}$$

$$\beta = (\beta_1, \beta_2, \dots, \beta_j, \dots) \tag{3}$$

where it is assumed that the higher the value of the suffix, the higher the technology level. Each firm occupies one level of α and one level of β at a time.

We classify all the firms according to their technology levels. Then the size of capital, labor, and product of all firms having the common pair of levels α_i and β_j can be denoted by K_{ij}, L_{ij}, and Q_{ij}, respectively. We will focus our attention to the time evolution of $\{K_{ij}\}$, and $\{L_{ij}\}$; where $\{Q_{ij}\}$ can be determined by the relation (1).

In order to obtain the equations describing the evolution of K_{ij} and L_{ij}, we consider an analogy between firms and genes. Technological level of a firm corresponds to genotype of a species; each size of K_{ij} and L_{ij} is comparable with the biomass of the population of the gene having the pair of genotypes of α_i and β_j.

In the field of population genetics, Matsuda and Ishii (1980) have proposed a replicon model to represent a general formulation of stochastic time evolution of replicons. Genes, organisms, and population of the same genotype are examples of replicons. In the same sense, we take as a replicon the assemblage of firms having the same technology level. Then we can adopt the replicon model for describing the dynamics of $\{K_{ij}\}$ and $\{L_{ij}\}$.

Following the replicon model, the evolution process has the two components. One is that of a growing process, which can be characterized by a growth rate, such as the Malthusian parameter. The other is that of a mutation process, which can be characterized by the mutation rate representing the probability of the transition from a pair of levels to another pair per unit time. We denote by m_{ij} and n_{ij} the Malthusian parameters of capital and labor having (α_i, β_j), respectively. Furthermore, we denote by μ_{ij}^{kl} and ν_{ij}^{kl} the mutation rate of transition from (α_i, β_j) to (α_k, β_l) for the capital K_{ij} and the labor L_{ij}, respectively.

Using these quantities we can describe generally the time evolution of K_{ij} and L_{ij} as follows:

$$\dot{K}_{ij} = m_{ij}K_{ij} + \Sigma_{kl}(\mu_{kl}^{ij}K_{kl} - \mu_{ij}^{kl}K_{ij}). \tag{4}$$

$$\dot{L}_{ij} = n_{ij} L_{ij} + \Sigma_{kl}(\nu_{kl}^{ij} L_{kl} - \nu_{ij}^{kl} L_{ij}). \qquad (5)$$

B. Simplified Case

In the above part we have constructed a general formulation of evolutionary process of firms. In the following part of this section, we assume the simplest form for the production function

$$Q = \min(\alpha K, \beta L), \qquad (6)$$

which is usually called as the Leontief type. Furthermore, we assume that there is abundant potential labor and, as a result, the amount of product is determined by capital as

$$Q = \alpha K. \qquad (7)$$

The amount of needed labor is assumed determined by capital from

$$L = Q/\beta = \alpha K/\beta. \qquad (8)$$

From (7) and (8) only K is an independent variable. Then the general formulation becomes simplified as

$$\dot{K}_i = m_i K_i + \Sigma_k(\mu_k^i K_k - \mu_i^k K_i). \qquad (9)$$

Here we must determine the forms of growth rate and mutation rate for capital. The profit P_i of i-th replicon would be

$$P_i = \text{total revenue} - \text{total cost} = pQ_i - (wL_i + rK_i) \qquad (10)$$

where p is the product price, w the wage rate, and r the capital cost.

A fraction of the profit P_i will be invested to increase the size K_i, which will be denoted by

$$kP_i,\ 0 \le k \le 1. \qquad (11)$$

Then, the growth rate m_i can be given as follows:

$$m_i = kP_i/K_i. \qquad (12)$$

When a firm having the routine α_i tries to improve technology through innovation, the probability of transition to a new routine α_k can be assumed to be constant as first-order approximation. On the other hand, when a firm tries to imitate advanced routines already adopted by other firms, the probability of transition to the new routine may not be constant but roughly proportional to the size of the firms having the routine. This is true because the outflow of information about the technology α_k becomes easier as K_k becomes larger. Then the rate may be written as

$$\mu_i^k = \mu_0 + \mu_1 x_k \tag{13}$$

where μ_0 and μ_1 are constants and

$$x_k = K_k/\Sigma_k K_k. \tag{14}$$

C. Analytical Properties for the Simplified Case

The Malthusian parameter defined by (12) becomes

$$m_i = k(p\alpha_i - w\alpha_i /\beta - r) \tag{15}$$

where (10) is adopted.

The total capital size of firms under consideration is given as

$$K = \Sigma_i K_i. \tag{16}$$

Summing up both sides of (9) for all α_i, we have the following equation:

$$\dot{K} = \overline{m} K \tag{17}$$

where

$$\overline{m} = \Sigma_i m_i K_i / \Sigma_i K_i. \tag{18}$$

Here we introduce average level of $\{\alpha_i\}$ as

$$\overline{\alpha} = \Sigma_i \alpha_i K_i / \Sigma_i K_i. \tag{19}$$

As noted previously, when the level of technology becomes higher, the value of α_i also increases. That is, the coefficients $\{\alpha_i\}$ increase as a function of the suffix i. When technological progress continues, the value of α increases with time.

Substituting (15) and (19) into (18), we have

$$\overline{m} = \Sigma_i\ k(p\alpha_i - w\alpha_i/\beta - r))K_i/\Sigma_i K_i = k\{(p - w/\beta)\overline{\alpha} - r\}. \qquad (20)$$

Here the price p is determined by industry output Q ($=\Sigma_i Q_i$), given the product demand-price function, D(Q). Then we have

$$p = D(Q) = D(\Sigma_i\ \alpha_i K_i) = D(K). \qquad (21)$$

The wage rate w depends on the total labor required. Thus

$$w = w(L) = w(\Sigma_i\ \alpha_i K_i/\beta) = w(\overline{\alpha}\ K/\beta). \qquad (22)$$

The product demand-price function generally decreases as industry output increases. For simplicity, we adopt the following relation:

$$p = A/Q = A/\overline{\alpha}\,K \qquad (23)$$

where A is constant. Expression (23) means that the price is inversely proportional to the total product produced.

The wage rate may increase when a larger amount of total labor is needed as a result of increase in total output. We assume the following form:

$$w = W + BL = W + B\overline{\alpha}\ K/\beta \qquad (24)$$

where W and B are constants. Then (20) becomes

$$\overline{m} = k\{A/K - BK(\overline{\alpha}\ /\beta)^2 - W(\overline{\alpha}\ /\beta) - r\}. \qquad (25)$$

Substituting (23) to (15), we have

$$\dot{K} = k\{A - (W\overline{\alpha}\ /\beta + r)K - B(\overline{\alpha}\ /\beta)^2K^2\}. \qquad (26)$$

From Equation (26) we can say that initially the size increases at a constant rate kA. The rate of increase is assumed to become smaller as the size approaches saturation. The saturated level depends on the parameters A, B, W, r, and and $\overline{\alpha}\ /\beta$. As pointed out ealier in this section, the value of $\overline{\alpha}$ depends on time. If the value of B is equal to zero, Equation (26) becomes a logistic form. This coincides with the fact that growing process of a new technology can be described by the logistic model.

4. COMPETITION AMONG COMMODITIES

In this section we apply a generalized Lotka-Volterra model to the competition among commodities. We focus out attention to the subsystem of commodity and market illustrated in Figure 1. To describe mathematically formulate the interaction of consumers and commodities we can use the theoretical model describing the interaction of consumers and resources in an ecological community.

A. General formulation

Let us assume that there are n different commodities in a market. We denote the amount of commodities in the market as

$$\text{amount of commodities} = (X_1, X_2, \ldots, X_n) \tag{27}$$

Furthermore, we devide the whole consumers to m different subsystems according to their incomes, generations, or other characters appropriate for each study. Let us write purchasing power of each subsystem consumer as

$$\text{purchasing power of consumers} = (R_1, R_2, \ldots, R_m) \tag{28}$$

We can generally describe the time evolution of each amount of commodity as

$$\dot{X}_i = k_i X_i (\Sigma_j \mu_{ij} R_j - T_i), \tag{29}$$

where k_i means the fraction of profit which will be invested to increase the size of production of i-th commodity; μ_{ij} denotes the ratio of total purchasing power of j-th consumer class which is used to buy i-th commodity; and T_i denotes the cost which is necessary in the production of i-th commodity.

For the time evolution of $\{R_j\}$ we can have generally the following equation,

$$\dot{R}_j = r_j R_j (1 - R_j / K_j) - \Sigma_i \mu_{ij} X_i R_j \tag{30}$$

where K_j denotes the carrying capacity of j-th consumer, and r_j denotes the growth rate of purchasing power. The first term of the right hand side means growth of purchasing power when there is no expenditure, and the second term denotes the decrease of purchasing power caused by buying various commodities by j-th consumer.

We can have another type of equation to describe the time evolution of $\{R_j\}$,

$$\dot{R}_j = D_j - \Sigma_i \mu_{ij} X_i R_j, \tag{31}$$

where the first term to the right side means that there is constant income per unit time and the amount is denoted by D_j.

For simplicity's sake, we use only Equation (30) for the time evolution of $\{R_j\}$ in this paper.

B. Simplified Case

As a simplified case we assume there are only two commodities and one kind of consumer in a market. Then we have the following equations,

$$\dot{X}_1 = k_1 X_1(\mu_1 R - T_1) \tag{32}$$

$$\dot{X}_2 = k_2 X_2(\mu_2 R - T_2) \tag{33}$$

$$\dot{R} = rR(1 - R/K) - (\mu_1 X_1 + \mu_2 X_2)R. \tag{34}$$

Our interest here is whether the two commodities coexist in the market or only one of them can exist. And if the latter case holds, then we want to know the condition of survival of the commodity.

As an initial state we take the steady state under the condition of there being no second commodity in the market. The state is given as

$$X_1^0 = (1 - T_1/\mu_1)\, r/\mu_1, \; X_2^0 = 0, R^0 = T_1/\mu_1. \tag{35}$$

Thereafter small amount of the second commodity, which is denoted by δX_2, is introduced to the market. The time evolution of this is determined by the following equation,

$$\dot{\delta X_2} = k_2\mu_2(T_1/\mu_1 - T_2/\mu_2)\,\delta X_2, \tag{36}$$

which is obtained by using Equations (33), and (35) , with

$$X_1 = X_1^0 + \delta X_1,\ X_2 = \delta X_2, \text{ and } R = R^0 + \delta R. \tag{37}$$

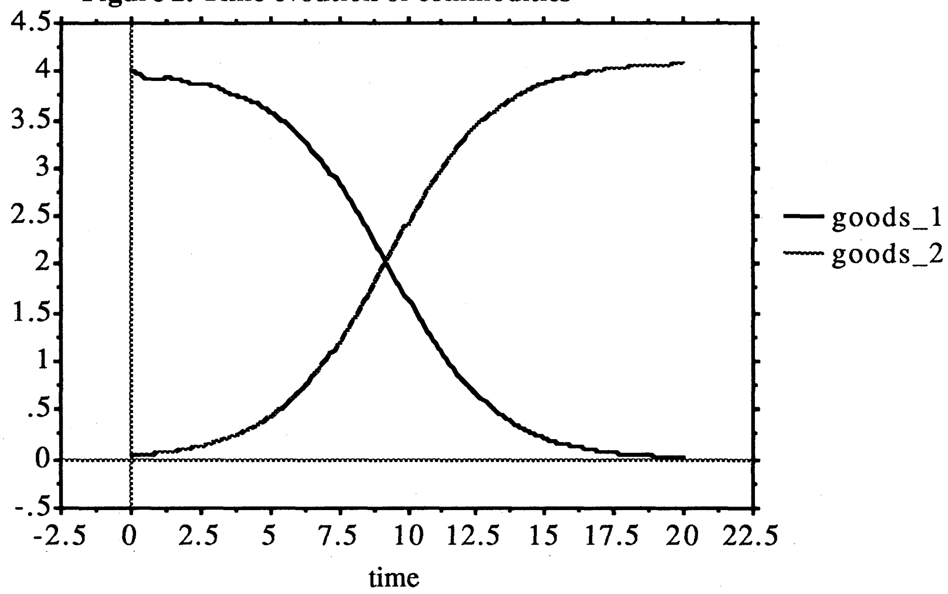
From this equation we can obtain the condition that the second commodity can penetrate to the market. This occurs when the right hand side of Equation (36) becomes positive value, that is when

$$T_1/\mu_1 > T_2/\mu_2. \tag{38}$$

We can interprete the term T/μ as an effective cost, so the condition (38) states that the second commodity can penetrate when its effective cost is smaller than that of the first commodity.

In Figure 2 is given the result of computer simulation, which shows the time evolution of both commodities in the market.

Figure 2. Time evoution of commodities



Here the values of the parameters are as follows:
k_1=0.5, T_1=10.0, μ_1=1.0, k_2=0.5, T_2=9.0, μ_2=1.0, r=5,0, K=50.0.

This corresponds to the condition (38), so the second commodity penetrates and the first commodity dissapears.

Let us consider the case of two commodities and two types of consummers in the market. We can have the following equations in this case,

$$\dot{X}_1 = k_1X_1(\mu_{11}R_1 + \mu_{12}R_2 - T_1), \tag{39}$$

$$\dot{X}_2 = k_2X_2(\mu_{21}R_1 + \mu_{22}R_2 - T_2), \tag{40}$$

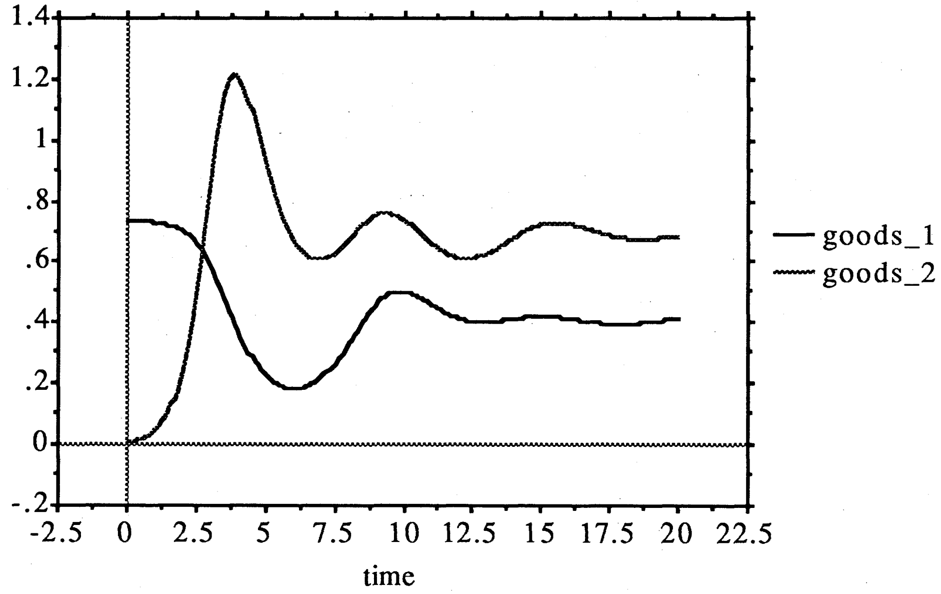
$$\dot{R}_1 = r_1R_1(1 - R_1/K_1) - (\mu_{11}X_1 + \mu_{21}X_2)R_1, \tag{41}$$

$$\dot{R}_2 = r_2R_2(1 - R_2/K_2) - (\mu_{12}X_1 + \mu_{22}X_2)R_2. \tag{42}$$

As in the prevous case, the initial state is given by the steady state under the condition of there being no second commodity. We are interested in the condition of invasion of second commodity to the market. Since there are two types of consumers in this case, it may be possible that the two commodities coexist in the market. We can have the condition of penetration of hte second commodity based on the similar method given to the previous case, but it is very complicated and is not given here.

We suppose if one class of consumer prefers the first commodity and the other class prefers the second commodity, then both commodities can coexist in the market.

Figure 3. Time evolution of commodities



This possibility can be verified by using computer simulation. The following values are selected for the parameters of Equations (39)-(42);

$k_1=0.5$, $T_1=5.0$, $\mu_{11}=0.8$, $\mu_{12}=0.2$, $k_2=0.5$, $T_2=4.0$, $\mu_{21}=0.2$, $\mu_{22}=0.8$, $r_1=1.0$, $K_1=10.0$, $r_2=1.0$, $K_2=10.0$.

The result of calculation is illustrated in Figure 3. Two commodities coexist in the market and show periodic time evolution.

5. DISCUSSIONS

We have shown that the basic theoretical models in ecology can be applied to describe various behaviors of economic system. Based on these theoretical models we can make computer simulations to analyze observed economic data.

A computer simulation was made for the replicon model, and the results were compared to the observed data for the cotton-spinning industry in Japan during 1956-1959 (Nishiyama, 1985). In these observations the measures of capital technology were approximated by the amounts of capital necessary for making unit amounts of production. The observed size distribution of the technology shows hill-shaped pattern, and the maximum peak moves to a higher technology level in a few years.

By comparing the observed distributions with those obtained from computer simulation, we had the conclusion that the observed distribution may indicate that there was rapid technological change by active imitative process during this period in this industry.

If there are other observed data concerning economic systems, we can analyze them by making computer simulation based on our theoretical models.

In this paper we formulated mainly the process of manufacturing. In addition to manufacturing industries, we should formulate the behaviors of service and information industries, because these industires are becoming more and more important and these need different theoretical model compared to the manufacture industry.

Evolutionary process of genetic information and brain may give clues to these theoretical formulation. Biological approach to economic system may become all the more important.

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